Conceptual Math
Algebra I
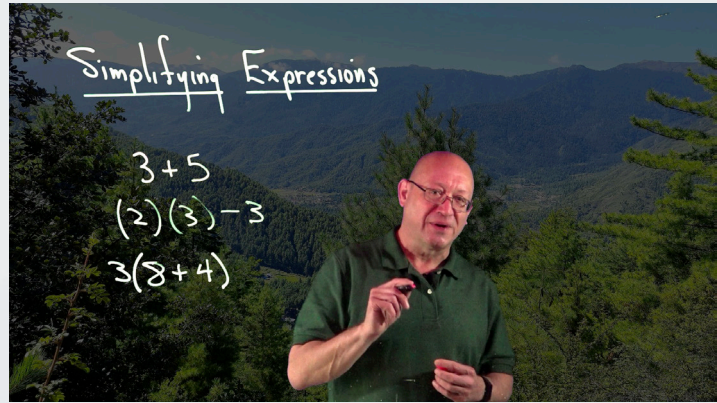
Chapter 4: Expressions

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Chapter 4

Expressions

4.1   Expressions

A critical skill in basic algebra is taking a given expression and making it as simple as possible using the properties of real numbers.

Arithmetic Expression – a sequence of literal numbers and operations.

Literal Number – a literal number is a number whose value is known. We express a literal number in the way we have since elementary school, by its value: 72, 5, 12, 213, –40, are all literal numbers.

An *arithmetic expression* is any sequence of literal numbers separated by operations.

$$4 + 3 \qquad 5 - 2 + 8 \qquad 7 + (6)(4) - 12 \qquad 32 + 24 \div 4$$

4.2   Simplifying Expressions

Simplifying an expression is performing as many operations as possible until no more can be done.

Consider the expression $6 + 5$

Can we write this as another expression that is simpler? YES. Just add the two numbers to get 11.

Consider $4 + 7 - 5$

We can add the 4 and the 7 to produce $11 - 5$

11 - 5 = 6

Consider $15 + 7 \cdot 3$

For the simplification of any expression, there are specific, precisely defined rules for the order in which operations are performed. There is no ambiguity.

Order of Operations in Expressions

At each level, we move from the left to the right.

Grouping symbols: parentheses, brackets, braces
Exponents
Multiplcation and Division
Addition and Subtraction

For operations that are at equal levels, we perform them LEFT TO RIGHT.

Examples

$4 \cdot 5^2 + 35 – 7 \cdot 3$

No grouping symbols. Highest precedent (what comes first) is the exponent.

$4 \cdot 25 + 35 - 7 \cdot 3$

Now we have two multiplications in the expression, the 4 times 25 and the seven times three.

Since these are equal in the hierarchy, we perform them left to right.

$100 + 35 - 21$

Now we have addition and subtraction, which have equal order, so we do them left to right.

$135 - 21$

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Consider:

$20 + (6 - 2)^2 - 4 \cdot 3$

Because we know exponents are high in the order, we might be tempted to do the exponent first, but grouping symbols are the top of the list. We must subtract the 2 from the 6, so our first step gives us:

$20 + 4^2 - 4 \cdot 3$

Now the exponent is the highest, so we get:

$20 + 16 - 4 \cdot 3$

Next comes the multiplication:

$20 + 16 - 12$

The addition and subtraction are equal, so we move left to right. $36 - 12$

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An **Algebraic Expression** is a sequence of numbers and operations that includes variables.

Examples:

$3x + 4$ $\qquad$ $5x^2 - 7x + 2$ $\qquad$ $4a^2 - 3ab$ $\qquad$ $x^5 + 4x$

As with arithmetic expressions, when algebraic expressions contain operations that can be performed, one can simplify them by performing the operation, resulting in an expression that has fewer operations. When no more operations are possible, the expression is considered simplified.

Definition: When an expression contains items separated by addition or subtraction, these items are called the _terms_ of the expression.

Expression = Term1 + Term2 - Term3 + Term4 - Term5

Example: $3xy + xy^2$ - $6x$

This expression has three terms: $3xy$, xy^2, and $6x$.

Example: $4a^3b + 5ab^2$ - $ab^5 + 17$

This expression has four terms: $4a^3b$, $5ab^2$, ab^5, and 17.

$$7x^2 + 3x^5y^3 - 4x^4y^7 + 6x^5y^6 - 21$$

TERMS

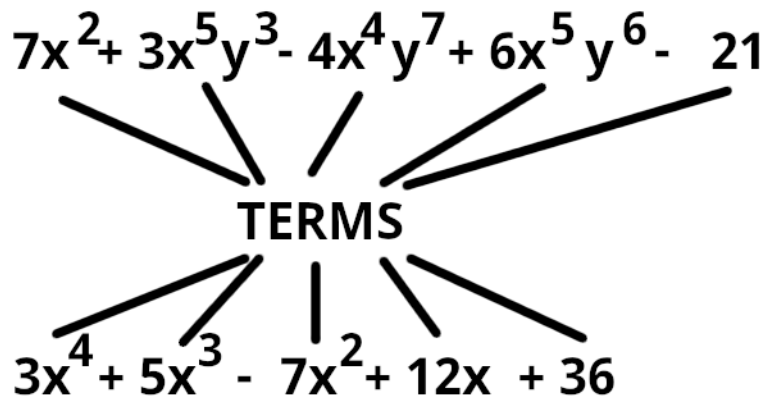$$3x^4 + 5x^3 - 7x^2 + 12x + 36$$

Figure 4.1: These expressions have five terms

Notice that the order of operations has addition and subtraction as the final

operations to be performed. Put another way, to simplify an expression, one simplifies EACH TERM prior to adding or subtracting them.

Definition: Two terms are _similar_ if they have the same variables raised to the same exponents. Two or more similar terms can be added or subtracted to produce a single term. This is often called "combining like terms."

NOTE: Literal numbers (also called constants) are similar since they have no variables. They can be added or subtracted, which students start doing in first grade.

Simplify: 3x + 7 - 2x + 4

The terms having an x variable and the two constant terms can be combined.

3x - 2x = x and 7 + 4 = 11, so we get:

x + 11

Simplify: $5x^2 + 3x - 2 + 4x^2 + 8$

The x^2 terms and the constant terms can be combined. There are no terms similar to 3x so it remains unchanged.

We get $9x^2 + 3x + 6$

Simplify: 32ab + 5a - 12b + $4a^2b^2$

This expression has no similar terms and cannot be simplified further.

Simplify: $6x^3 + x^2 - 2x + 4x^2 - 7 + 2x^3$

The terms with x^3 and x^2 can be combined. We get:

$8x^3 + 5x^2 - 2x - 7$