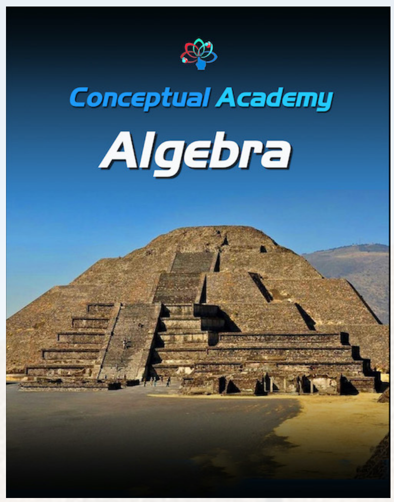
Conceptual Math
Algebra I

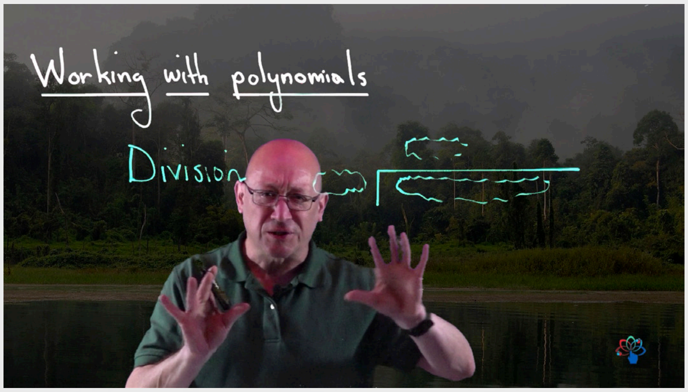
Chapter 5: Polynomials



Matt Foraker, Ph.D.
Western Kentucky University
Bowling Green, KY

Conceptual Academy

All inquiries
Support@ConceptualAcademy.com



Chapter 5

Polynomials

5.1 What's a Polynomial?

A polynomial is an algebraic expression that has the form

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0.$$

Some students find the subscript notation intimidating at first, but each a_i is a literal number that multiplies the term by the value of that number. These numbers are called <u>coefficients</u>.

Polynomial expressions have only one variable. Each term of a polynomial consists of a multiple (coefficient) of the variable raised to a positive integer exponent. Again, the number a_i is the coefficient of the term with x^i.

5.2 Adding and Subtracting Polynomials

Since polynomials are algebraic expressions, all of the principles associated with simplifying expressions apply to polynomials.

To add or subtract polynomials, combine like terms.

EXAMPLE ONE:

$(x^2 + 3x + 5) + (3x^2 - 7x + 2)$

For the x^2 term we have one and three, adding to four. For the x term, we have three and negative seven, which add to negative four. For the constant term, we have five and two, adding to seven. So the result is

$4x^2$ - $4x + 7$

EXAMPLE TWO:

$(x^3 + 6x^2 + 2x - 4)$ - $(5x^2 - 3x + 6)$

We only have one term for x cubed, so it remains unchanged. For x squared, we have six minus five, which is one. For the term with x, we have two minus a negative three, which is five. For the constant term, we have a negative four minus a positive six, which is negative ten.

$x^3 + x^2 + 5x - 10$

EXAMPLE THREE:

$(3x^5 + x^4 - 4x + 8) + (7x^4 - 7x^3 + 6x)$

We have one term for x^5 so it remains unchanged. For x^4 we have one and seven, adding to eight. There is no term for x cubed in the first polynomial, so we have what is in second one, a negative seven. Neither polynomial has a term for x squared. For x, we have a minus four added to a positive six, which is two. The only constant term is 8. This leaves us with:

$x^5 + 8x^4 - 7x^3 + 2x + 8$

5.3 Polynomial Multiplication and FOIL

Most young children in their arithmetic classes were taught how to multiply two numbers "by hand" using a procedure that recognized the power of ten raised to each digit and properly manipulated the digits to produce the correct result.

Remember that 1,284 is 1,000 (one 10^3), 200 (two 10^2), 80 (eight 10^1), and 4 (four 10^0). Each digit represents the quantity the number contains of each power of ten.

Many years ago, in elementary school students learning math would see numbers represented as sticks. When the number of sticks reached ten, they were tied into a bundle. Ten bundles together were grouped into a ”big bundle” (100), and ten ”big bundles” together made a ”super big bundle” (1000), and so on. This is base 10. Each digit represented a power of ten.

In many ways, polynomial expressions match the composition and behavior of numbers with a given base, but with polynomials the base is the variable, and the digits are the coefficients. One result is that multiplying polynomials is very much like the multiplication taught to young children.

EXAMPLE:

$$\begin{array}{r}
x^2 + 4x + 4 \\
\underline{\qquad\qquad 2x + 1} \\
2x^3 + 8x^2 + 8x \qquad \\
\underline{\qquad x^2 + 4x + 4} \\
2x^3 + 9x^2 + 12x + 4
\end{array}$$

TRY THIS: Multiply 144 $\times$ 21 by hand. Notice anything? Let x = 10 in $2x^3 + 9x^2 + 12x + 4$ and calculate.

Polynomials with two terms are called binomials. To multiply two binomails, we use an algorithm known and remembered by virtually all algebra: FOIL. FOIL is an acronym that stands for First, Outer, Inner, Last. Each of these designates two terms to be multiplied, leading to four products. The sum of the four products is the result of the multiplication of the two binomials.

F O I L

F O I L

Consider the product (A + B)(C + D)

FIRST: The product of the first term in each binomial - AC.
OUTER: The product of the "outer" terms - AD.
INNER: The product of the "inner" terms - BC.
LAST: The product of the second term in each binomial - BD.

The product of (A + B)(C + D) = AC + AD + BC + BD.

Students do not memorize and master the above line with capital letters. They memorize the process and practice until it becomes as automatic as brushing one's teeth or tying a shoe.

EXAMPLE: (x + 4)(x + 3)

First + Outer + Inner + Last
(x)(x) + (x)(3) + (4)(x) + (4)(3)

$x^2 + 3x + 4x + 12$

$x^2 + 7x + 12$

EXAMPLE: (2x + 5)(x - 4)

(2x)(x) + (2x)(-4) + (5)(x) + (5)(-4)

$2x^2 - 8x + 5x - 20$

$2x^2 - 3x - 20$

EXAMPLE: (4x - y)(x + 3y)

(4x)(x) + (4x)(3y) + (-y)(x) + (-y)(3y)

$4x^2 + 12xy - xy - 3y^2$

$4x^2 + 11xy - 3y^2$

The multiplication of binomials and becoming comfortable with FOIL is an important skill in algebra, particularly when working with quadratic equations (covered in Chapter 16). Multiplying and dividing polynomials by hand is not part of this course.

We will address dividing higher order polynomials by binomials when we learn to find the zeros of polynomials. We will learn synthetic division in the context of finding zeros and graphing polynomials (covered in section 1.2.3).