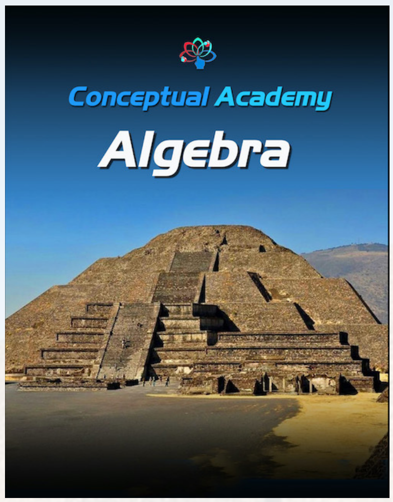
Conceptual Math
Algebra I

Chapter 6: Factoring Expressions and Polynomials



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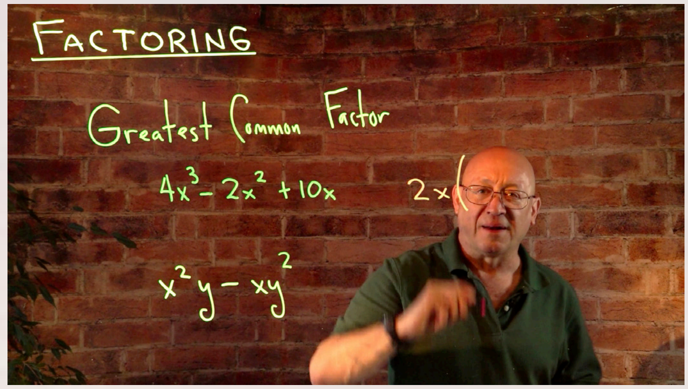

Chapter 6

Factoring

6.1 What is Factoring?

You have likely encountered factoring in an elementary math class where you were asked to find the numbers that multiply together to produce a given number. For example, the numbers that multiply to produce 96 are $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$.

The ability to break a product into the numbers multiplied together to produce it was useful to find least common denominators when adding or subtracting fractions.

With algebraic expressions containing variables, factoring involves recognizing the situations that can occur when various expressions are multiplied together.

(PRODUCT) = (factor 1)(factor 2)(factor 3) . . .

EXAMPLE:

$x^3y^2 - 3x^2y = x^2y(xy - 3x)$

The above is an example of recognizing that each term has a common multiple which can be "factored." Some find it useful to think of this as using the Distributive Law "in reverse." Applying the Distributive Law to the result is a way of confirming that one factored the expression correctly.

TIP: Apply the distributive law to simplify $x^2y(xy - 3x)$

NOTE: Factoring is NOT simplifying!!

EXAMPLE:

$x^2 - 3x - 4 = (x + 1)(x - 4)$

One can use FOIL to confirm (try it!).

So how does one factor algebraic expressions? For our purposes, there are four situations to address that become relevant when solving equations and graphing functions.

6.2 Factoring Expressions

When factoring algebraic expressions, we most often encounter one of the following four situations:

1. Greatest common factor
2. Trinomials
3. Perfect Squares
4. Difference of Squares

Remember: factoring is breaking up a product of expressions into the separate factors that were multiplied together to produce it.

A literal numbers: $180 = 2 \cdot 90 = 2 \cdot 2 \cdot 45 = 2 \cdot 2 \cdot 3 \cdot 15 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$.

Algebraic expression: $= x^2 + 5x + 6 = (x + 2)(x + 3)$

6.2.1 Greatest Common Factor

When you have an expression where the terms have a common factor.

$4x^3 - 2x^2 + 8x = (2x)(2x^2 - x + 4)$

$x^2y - xy^2 = xy(x - y)$

6.2.2 Trinomials

Trinomials are polynomials of the form $ax^2 + bx + c$ where a, b, c are literal numbers. Some trinomials can be factored, i.e. they are the product of two monomials $(qx+r)(tx+v)$ for some numbers q, r, t, v.

Example: Factor $x^2 + 5x + 6$

To do this we think of FOIL in reverse and we get $(x+2)(x+3)$.

THINK: Why does $(x+1)(x+6)$ fail?

$x^2 + 10x + 21 = (x+7)(x+3)$
$x^2 - 7x + 12 = (x-4)(x-3)$
$2x^2 + 13x + 15 = (2x+3)(x+5)$
$3x^2 + 10x - 8 = (3x-2)(x+4)$

6.2.3 Perfect Squares

Recognizing Perfect Squares of the form $(A + B)^2 = A^2 + 2AB + B^2$ where A, B, and C can be any expression.

$(x+1)^2 = (x+1)(x+1) = x^2 + x + x + (1)(1) = x^2 + 2x + 1$

$(x+2)^2 = (x+2)(x+2) = x^2 + 2x + 2x + (2)(2) = x^2 + 4x + 4$

$(x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + (3)(3) = x^2 + 6x + 9$

$(x+4)^2 = (x+4)(x+4) = x^2 + 4x + 4x + (4)(4) = x^2 + 8x + 16$

$(x+5)^2 = (x+5)(x+5) = x^2 + 5x + 5x + (5)(5) = x^2 + 10x + 25$

$(x+6)^2 = (x+6)(x+6) = x^2 + 6x + 6x + (6)(6) = x^2 + 12x + 36$

$(x+7)^2 = (x+7)(x+7) = x^2 + 7x + 7x + (7)(7) = x^2 + 14x + 49$

$(x+8)^2 = (x+8)(x+8) = x^2 + 8x + 8x + (8)(8) = x^2 + 16 + 64$

$(x+9)^2 = (x+9)(x+9) = x^2 + 9x + 9x + (9)(9) = x^2 + 18x + 81$

$(x+10)^2 = (x+10)(x+10) = x^2 + 10x + 10x + (10)(10) = x^2 + 20x + 100$

Just be able to recognize them when you see them and instantly write down the result.

6.2.4 Differences of Squares

These are VERY easy to see. First, the situation always involves TWO terms where the second is subtracted from the first. In the factoring context, if you see two terms with a subtraction, immediately examine the terms to see if they are the result of squaring something. If so, you have a difference of squares.

For any expressions A and B,

$A^2 - B^2 = (A + B)(A - B)$

$x^2 - 4 = (x+2)(x-2)$
$x^2 - 25 = (x+5)(x-5)$
$x^2 - 36y^2 = (x+6y)(x-6y)$
$9x^2 - 64 = (3x+8)(3x-8)$
$a^2b^4 - w^2 = (ab^2+w)(ab^2-w)$

6.2.5 Factoring Guidelines

1. Examine each term. Is there a common factor that can be removed? If so, factor it out ”in front” and continue working on what is left.

2. Is it a trinomial? Draw two sets of parentheses with space between them and apply the ”reverse FOIL” trial and error process until you find the two binomials that produce the trinomial.

3. Is it a perfect square? If so, write the factored result.

4. Is it a difference of squares? If so, write the factored result.

We require the above to perform important tasks later on.