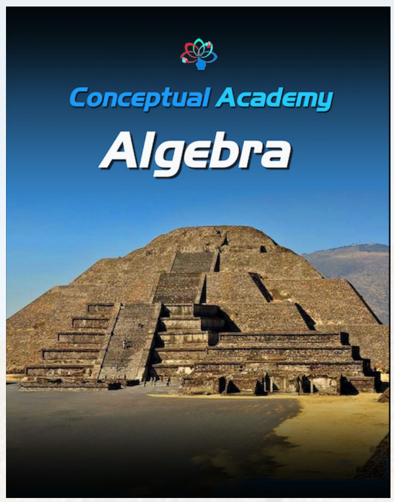
Conceptual Math
Algebra I

Chapter 7: Rational and Radical Expressions



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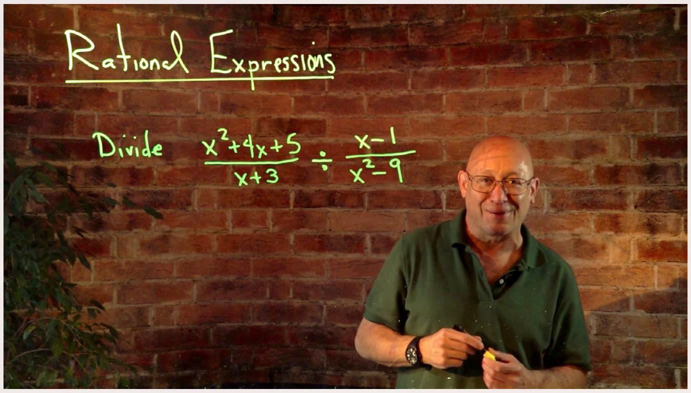

Chapter 7

Rational and Radical Expressions

7.1 Rational Expressions

Rational expressions have the form $\frac{A}{B}$ where A and B are polynomials.

We add, subtract, multiply, and divide rational expressions using the same principles used when working with fractions.

To multiply two fractions: $\left(\frac{A}{B}\right)\left(\frac{C}{D}\right) = \frac{AC}{BD}$

To divide two fractions – invert the second fraction and multiply

$$\frac{A}{B} \div \frac{C}{D} = \left(\frac{A}{B}\right)\left(\frac{D}{C}\right) = \frac{AD}{BC}$$

To add or subtract two fractions, one must obtain a common denominator and then add or subtract the numerators.

Consider $\frac{1}{3} + \frac{1}{2}$

Here the common denominator is 6, so we multiply each fraction by the fraction that produces a denominator of 6.

$\frac{1}{3}\left(\frac{2}{2}\right)+\frac{1}{2}\left(\frac{3}{3}\right)$

$\frac{2}{6}+\frac{3}{6}$

$\frac{5}{6}$

Let's do $\frac{2}{3}$ - $\frac{1}{4}$

$\frac{2}{3}\left(\frac{4}{4}\right)$ - $\frac{1}{4}\left(\frac{3}{3}\right)$

$\frac{8}{12}$ - $\frac{3}{12}$

$\frac{5}{12}$

Do the laws change when we have algebraic expressions with variables?

NO!!!

Consider $\frac{x}{3}+\frac{1}{5}$

Common denominator is what? 15!

$\frac{5x}{15}+\frac{3}{15}=\frac{3x+5}{15}$

Simplify $\frac{4}{x}+\frac{x}{3}$

Common denominator? 3x

$$\frac{12}{3x} + \frac{x^2}{3x}$$

$$\frac{x^2 + 12}{3x}$$

Simplify $\quad \frac{1}{x} + \frac{1}{y}$

Common denominator? $\quad$ xy

$$\frac{y}{xy} + \frac{x}{xy}$$

$$\frac{x + y}{xy}$$

Simplify $\quad (\frac{x}{y})(\frac{2x}{yz})$

$$\frac{2x^2}{y^2 z}$$

Add $\frac{x+1}{x-2} + \frac{x+3}{x+2}$

What is the common denominator? We have a common denominator that is a product of the two.

$$\frac{(x+1)(x+2)}{(x-2)(x+2} + \frac{(x+3)(x-2)}{(x-2)(x+2)}$$

$$\frac{(x^2+3x+1)}{(x^2-4} + \frac{x^2+x-2)}{(x^2-4}$$

Simplify:

$$\frac{x^2+x-2}{x+5} \cdot \frac{x^2-25}{x-2}$$

$$\frac{(x+3)(x-2)}{x+5} \cdot \frac{(x+5)(x-5)}{x-2}$$

The (x - 2) and (x + 5) occur in both numerator and denominator and cancel, leaving us with:

$(x+3)(x-5) = x^2 - 2x - 15$

Rational expressions can look complicated and cumbersome, and a trap for beginning students is to become intimidated and lose the confidence to continue. With practice one realizes the expressions are not complex and that applying the same rules as for other fractions allows one to perform the operations.

$$\frac{x^2+6x–7}{x+3} \div \frac{x-1}{x^2-9}$$

$$\frac{(x+7)(x–1)}{x+3} \div \frac{x-1}{(x+3)(x-3)}$$

$$\frac{(x+7)(x–1)}{x+3} \cdot \frac{(x+3)(x-3)}{(x-1)}$$

$(x+7)(x-3)$

$x^2 + 4x - 21$

7.2 Radical Expressions

Radical expressions are expressions that contain radical signs. Simplifying a radical expression involves the same principles as simplifying other expressions with the added requirement that each radical is simplified as discussed in section 3.7. In that section we addressed only an isolated radical sign.

Here we consider more complicated expressions that can include several radicals. Note that the rules for simplification remain the same.

NOTE: Many students find it easier to replace radical signs with rational exponents (remember that radicals are fractional exponents!) and then working with the rules for exponents. This is a matter of preference.

EXAMPLE: Simplify $(\sqrt{x^3y^2})^3$

From the properties we know that this is $\sqrt{x^9y^6}$

Which gives us $x^4y^3\sqrt{x}$.

EXAMPLE: Simplify $(\sqrt{x^4y^2})^2(\sqrt{x^3y})^5$

Let's write this one using rational exponents.

$(x^4y^2)^{\frac{1}{2}}(x^3y)^{\frac{1}{5}}$

From the properties of exponents this is:

$(x^2y)(x^{\frac{3}{5}}y^{\frac{1}{5}}) = x^2y(x^3y)^{\frac{1}{5}} = x^2y\sqrt[5]{x^3y}$

EXAMPLE: Simplify $\sqrt[3]{a^6b^2}$

We get $a^2b^{\frac{2}{3}} = a^2\sqrt[3]{b^2}$

EXAMPLE: Simplify $(\sqrt{w^3z^4})^5(\sqrt[4]{w^7z^3})^3$

From the properties we get:

$\sqrt{w^{15}z^{20}}\sqrt[4]{w^7z^9}$

The first radical simplifies to $w^7z^{10}\sqrt{w}$.
The second radical simplifies to $wz^2\sqrt[4]{w^3z}$ which gives us:

$(w^7 z^{10} \sqrt{w})\ (w z^2 \sqrt[4]{w^3 z})$ or

$(w^8 z^{12} \sqrt{w})\ (\sqrt[4]{w^3 z})$.

Can the square root and the fourth root be combined? YES.

This is easily seen when written with rational exponents or one can recognize that $\sqrt{w}$ is the same as $\sqrt[4]{w^2}$. Using either we obtain:

$w^8 z^{12} \sqrt[4]{w^5 z}$

or

$w^9 z^{12} \sqrt[4]{wz}$.

This last example provides a taste of how working with radicals can be cumbersome and error prone. Fortunately one can eliminate this by writing the expression using rational exponents. The properties of exponents are easier to work with and as they are very important, students must learn the rules for working with exponents and will therefore know them.