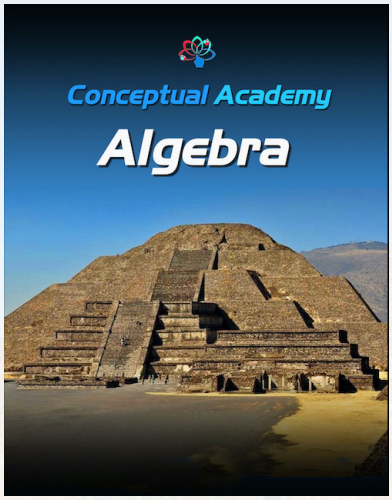
Conceptual Math
Algebra I

Chapter 8: Equations



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Chapter 8

Equations

8.1 Equations

Remember that an algebraic expression is simply a sequence of numbers and operations. You have a number, then an operation followed by another number, and this continues for the length of the expression. Note that there is no equal sign here. There is nothing to SOLVE.

The task we have with dealing with expressions is to SIMPLIFY them. The other task we have covered is to FACTOR a product into the factors that multiply to produce it.

Note that neither simplifying nor factoring considers the values of the variables. They are real numbers that can have any value.

Something very important happens when we introduce an equal sign = and state that two different expressions are equal.

THE EQUATION

EXPRESSION = ANOTHER EXPRESSION

While it is possible for an equation to be true (both sides are equal) regardless of the values of the variables, it is far more common for the equation to be true ONLY if the variables have certain values.

Consider 2x = 6.

There is only one value for x where this statement will be true. We have x = 3.

The value(s) of the variables that make the equation true are called the SOLUTION of the equation. The process of finding the solution is SOLVING the equation.

An equation can have more than one solution. It is possible for an equation to have an infinite number of solutions.

EXPRESSIONS: Simplify (perhaps factor)

EQUATIONS: Solve.

8.2 Solving Equations

Consider: $x^2 = 16$.

What value of x is such that this equation will be true? The answer is four, but is that the only solution? Is there any other number that produces 16 when it is squared? Yes, negative four also works.

The solution is x = 4 and -4.

DEFINITION: To SOLVE an equation is to find its solution.

To SOLVE an equation is to find the set of values for the variables in the equation that make the equation true.

FACT: An equation can have no solution, one solution, or more than one solution. It is possible for an equation to have an infinite number of solutions.

Consider the equation: $17 - 3x = 5$.

When this simple, we can apply reason to find the solution. What number subtracted from 17 will result in 5? The answer is 12. Well, three times what is 12? We get 4. The solution is x = 4.

How about the equation $x + 2y = 12$?

This equation has two variables. Remember the solution of an equation is the set of values of the variables that make the equation true. In this case we need a value for x and a value for y that works.

Suppose we let x = 2. What value of y would make this equation true?

If we set x = 2, we get 2 + 2y = 12.

What do we have to add to 2 to get 12?

10. So 2y must be 10 for this to work. Two times what is 10? Five!

{ x = 2, y = 5 } is a solution. } Is that the only solution?

Suppose we let x = 6?

This leads to 6 + 2y = 12, so 2y must be 6. Two times what is 6?

We get 3. So, { x = 6, y = 3 } is a solution to the equation.

We'll discuss equations of this kind in a later lesson, but for now know that there are an infinite number of solutions to this equation. In fact, for this equation there is a value of y that will work for ANY value of x, and vice versa. More about this later.

8.3 The Principles of Equality

Properties of Equality

If A = B then A + C = B + C and A – C = B – C

If A = B then AC = BC and $\frac{A}{C} = \frac{B}{C}$

Capital letters represent expressions. C can be more than just a number or variable. C could be $2x + 3$ or $6x^2 + 5$.

We SOLVE (find the solution) a linear equation by performing any of the four operations on BOTH sides of the equation with the goal of isolating the variable on one side by itself. What is on the other side is the solution of the equation.

IMPORTANT: Always simplify BOTH sides of an equation before attempting to solve it.

Solving linear equations:

1. Simplify both sides of the equation. 2. Apply the properties of equality to isolate the variable on one side of the equation.

INSERT MORE EQUATIONS HERE.

Solve: $3(4x + 2) - 6x = 30$

Simplify both sides.

$12x + 6 - 6x = 30$

$6x + 6 = 30$

$6x = 24$

$x = 4$

Solve: $(4x + 6) - 3x = 4x + 12$

$x + 6 = 4x + 12$

$6 = 3x + 12$

Or $-6 = 3x$

Or $x = -2$

Solve: $2(3x + 1) - 3(x - 7) = 4(x - 2)$

REMEMBER: Simplify both sides before starting any operations!!

$6x + 2 - 3x + 21 = 4x - 8$

$3x + 23 = 4x - 8$

$3x + 31 = 4x$

$31 = x$