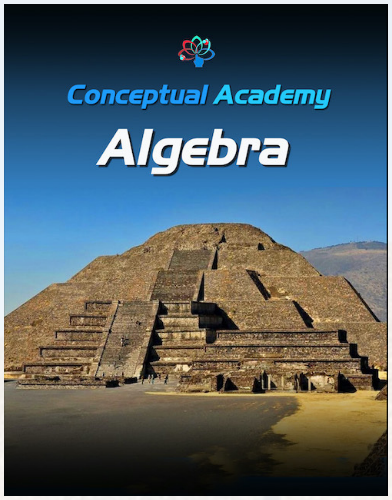
Conceptual Math
Algebra I

Chapter 10: Linear Inequality, Absolute Value, and Radical Equations



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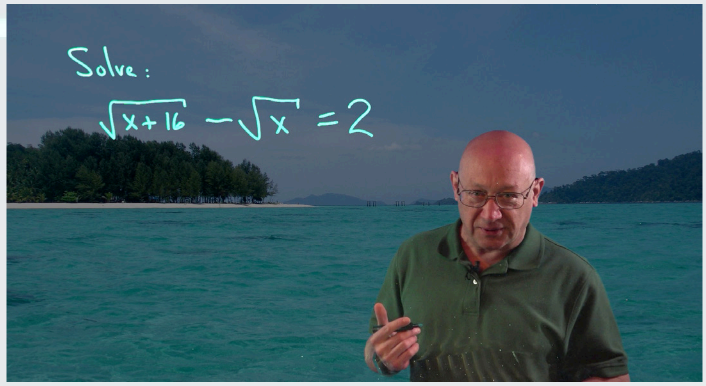
Chapter 10



Linear Inequality, Absolute Value, and Radical Equations

10.1 Linear Inequality

A linear inequality in one variable has the form

$ax + b > 0$ *or* $ax + b < 0$

We can also have "or equal" in the inequality

$ax + b \geq 0$ *or* $ax + b \leq 0$

The principles of equality also apply to inequalities, so we perform the same procedure to isolate the variable and solve the equation. One difference is that the solution contains an inequality sign instead of the equal sign, so we likely get a solution that consists of a range of values.

Another difference is that we must reverse the direction of the inequality whenever both sides of the equation are multiplied or divided by a negative number.

Solve: $2x + 5 < 5x – 7$

$5 < 3x – 7$

$12 < 3x$

$4 < x$

Some consider it better to express the solution with the variable occurring first: $x > 4$.

Solve: $\dfrac{2-t}{6} \geq 20$

$2 - t \geq 120$

$-t \geq 118$

$t \leq 118$

Compound inequalities contain more than one inequality sign.

Solve: $6 < 3x + 5 < 20$

$1 < 3x < 15$

$\dfrac{1}{3} < x < 5$

Solve $-4 < 5 - \dfrac{4}{5}x < 6$

$-9 < -\dfrac{4}{5}x < 1$

$-45 < -4x < 5$

$45/4 > x > -5/4$

BEST PRACTICE: When solving compound inequalities of this kind, always check to see that the numbers are consistent with the inequality direction. In the above we have 45/4 greater than x which is greater than -5/4. If we had the first number less than the second, something has gone wrong.

10.2 Absolute Value Equations

Let's remind ourselves that $\mid x \mid$ is the absolute value of x. On the real number line, one can consider the absolute value of a number as its distance from zero. A distance is always positive.

$\mid 5 \mid = 5 \qquad \mid -4 \mid = 4 \qquad \mid 0 \mid = 0$

When we have the absolute value of an expression in an equation, it is necessary to *convert or translate* the given equation into an equivalent equation or equations that do not have absolute values.

For any expression $ax + b$ with real numbers a and b, we have:

$\mid ax + b \mid = k$ translates to $ax + b = -k \;\; OR \;\; ax + b = k$

$\mid ax + b \mid < k$ translates to $-k < ax + b < k$

$\mid ax + b \mid > k$ translates to $ax + b < -k \;\; OR \;\; ax + b > k$

In the first situation, the absolute value of the expression is equal to k, so its value has a distance equal to k from zero. This is either k or -k. In the second situation, the distance from zero is *less than k*. This produces the inequality shown. In the third case, the distance from zero is *greater than k*, so we have the two inequalities. Note that greater than k or less than -k has a distance greater than k from zero.

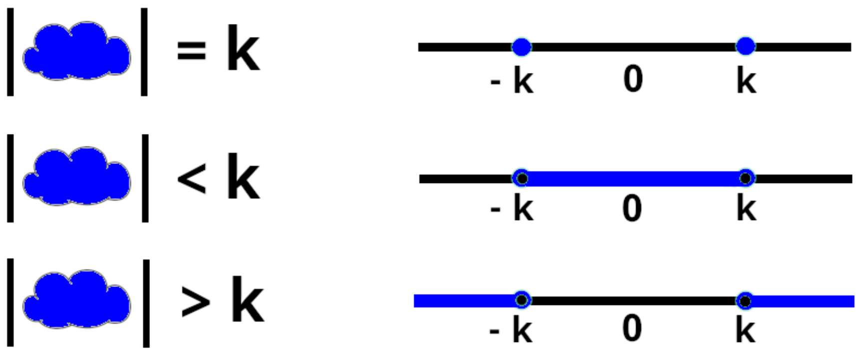


Figure 10.1: Absolute Value Translations

We solve equations with absolute value by re-writing the equivalent equations.

Solve $\mid 3x + 5 \mid= 17$

$3x + 5 = –17$ OR $3x + 5 = 17$

$3x = –22$ OR $4x = 12$

$x = \frac{22}{3}$ OR $x = 3$

Solve $\mid 7x–11 \mid< 32$

$–32 < 7x–11 < 32$

$–21 < 7x < 43$

$–3 < x < 43/7$

Solve $\mid 6x + 13 \mid > 25$

$6x + 13 < -25$ OR $6x + 13 > 25$

$6x < -38$ OR $6x > 12$

$x < -19/3$ OR $x > 2$

10.3 Radical Equations

A radical equation is an equation that contains radical expressions.

Solve $\sqrt{x} = 11$

The equation isn't asking us to find a square root. It's asking what number has a particular square root, in this case 11. When it's this simple, if we understand square roots we simply look at the equation and say, "OK, what's 11 squared? We get 121, and we have our answer of x = 121.

If the equation gets more complicated and we not able to just answer it by inspection, we use a process to eliminate the radical signs and write an equivalent equation we can solve by methods we already have.

We do this by recognize that if we raise a root to the power involved, the radical disappears. If we square a square root, cube a cube root, and so on, we are left with what was under the radical sign by definition.

For any x, we know that $\sqrt{x^2} = x$.

Similarly, we know that $\sqrt[3]{x^3} = x \; and \; \sqrt[4]{x^4} = x$.

We solve equations with radical expressions by isolating them and raising both sides of the equation to the power that eliminates them.

Example $\sqrt{x+3}+2=6$

First, we isolate the radical by subtracting the two.

$\sqrt{x+3}=4$

Squaring both sides eliminates the radical sign.

x + 3 = 16.

Subtract 3.

x = 13

The procedure does not change if the variable shows up more than once.

Solve $\sqrt{6x} \ - x \ = 2$

$\sqrt{6x} \ = x+2$

$6x \ = x^2+2x+4$

$0 \ = x^2-4x+4$

$0 \ = (x-2)^2$

$x \ = 2$