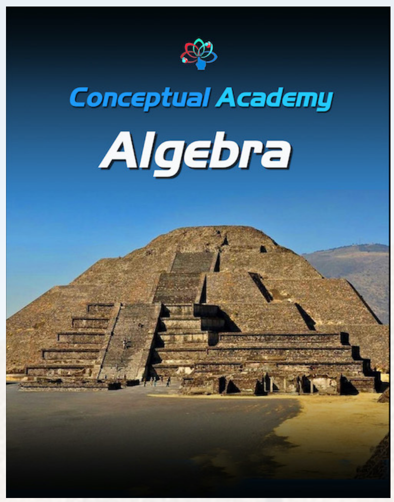
Conceptual Math
Algebra I

Chapter 11: The Coordinate Plane



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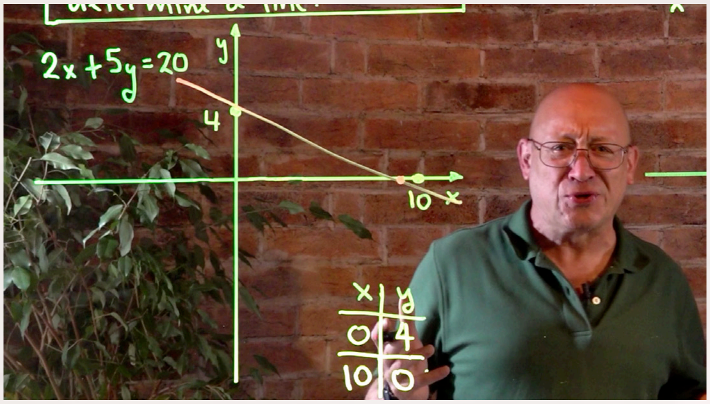

Chapter 11

The Coordinate Plane

Remember that an equation is a statement that sets two expressions equal.

EXPRESSION A = EXPRESSION B

And that the solution of an equation consists of the sets of values of the variables that make the equation true.

Consider the equation with variables x and y that has the form $ax + by = c$.

Say we have 3x + 2y = 12

The solution of this equation will be values of x and y that make the equation true.

If we let x = 4 and y = 0, the left hand side will be 12, making the equation true. *One* solution is x = 4 and y = 0.

Is that the only solution? Let's try x = 0 and y = 6. That also works. *Another* solution is x = 0, y = 6.

What about x = 2 and y = 3? That is a third solution. It turns out that this equation has an infinite number of solutions, each consisting of a value for x and a value for y that satisfy the equation.

A set consisting of a value for x and a value for y is called an ordered pair.

An ordered pair is denoted by (x, y) in parentheses.

The three solutions we have identified are (4,0), (0,6) and (2,3).

TRY THIS: Confirm that (-2, 9) is also a solution to the $3x + 2y = 12$.

NOTE: Again, the equation is expressing a RELATIONSHIP between the variables x and y. Whatever value we set for x, the value of y is going to ADJUST so that the equation continues to be true.

There are an infinite number of pairs (x,y) that solve this equation. To express the solution of equations of this kind, we **SHOW** the solution in the xy plane.

11.1 The XY Coordinate Plane and Ordered Pairs

A set consisting of a value for x and a value for y is called an ordered pair. An ordered pair is denoted by (x, y) in parentheses.

We express sets of ordered pairs using the xy coordinate plane. We create a horizontal line to graph the value of x and a vertical line to graph the value of y.

This produces a two-dimensional plane with $x = 0$ and $y = 0$ at the center. Any ordered pair can then be placed in the plane using the x and y values. The value of x is the x coordinate, and the point will have this horizontal distance from the $x = 0$ (the y-axis). The value of y is the distance of the point from $y = 0$ (the x-axis).

Figure 11.1 shows four ordered pairs plotted in the plane.

Returning to the equation from the last section, $3x + 2y = 12$, we showed that (4,0), (0,6), (2,3), and (-2, 9) were solutions. If we plot these points, notice that they fit along a straight line.

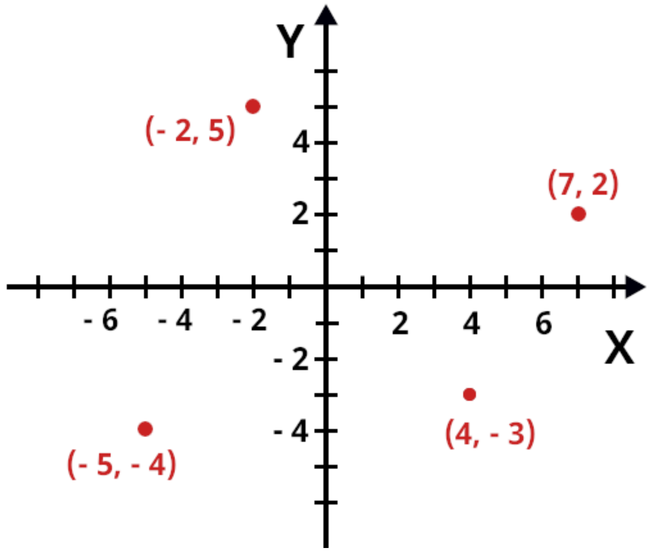

Figure 11.1: Ordered Pairs in the XY Plane

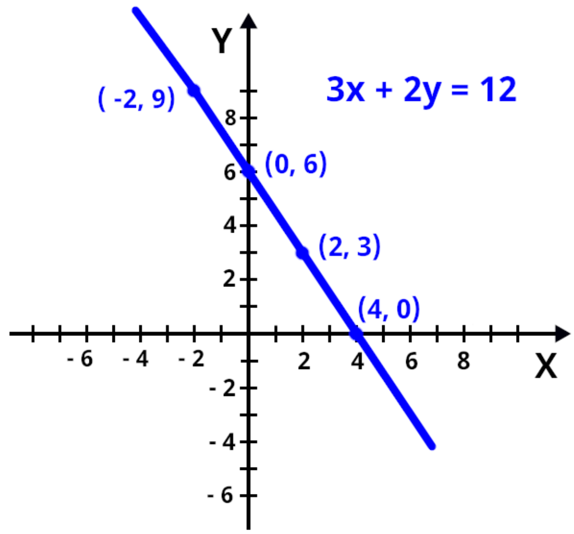

Figure 11.2: The graph of 3x + 2y = 12

11.2 Distance and Midpoint

11.2.1 The Distance Formula

The distance formula follows directly from the Pythagorean Theorem for right triangles. If you remember your geometry, you know that for a right triangle, the square of the hypotenuse is equal to the sum of the squares of the two adjacent sides.

The lengths of the adjacent sides are the distance between the two x-coordinates the two y coordinates.
The Let D = length of hypotenuse and we get $D^2 = (x_2 – x_1)^2 + (y_2 – y_1)^2$

Then take the square root, and we get the formula for the distance between two points.

D = $\sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$

Example: What is the distance between (1, 3) and (4, 7)?

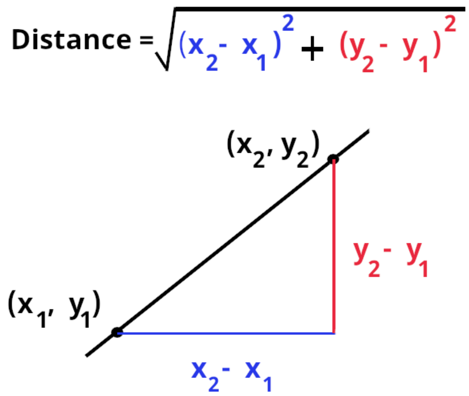

Figure 11.3: The Distance Formula

D = $\sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$
D = $\sqrt{(4–1)^2 + (7–3)^2}$
D = $\sqrt{3^2 + 4^2}$
$D = \sqrt{9 + 16}$
$D = \sqrt{25}$
D = 5

Example: What is the distance between (-2, 5) and (3, -7)?

D = $\sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$
D = $\sqrt{(3–(-2))^2 + (-7–5)^2}$
D = $\sqrt{5^2 + (-12)^2}$
$D = \sqrt{25 + 144}$
$D = \sqrt{169}$
D = 13

11.2.2 Midpoint

To find the midpoint between two points simply take the average of their x coordinates and their y coordinates. Expressed as a formula we get:

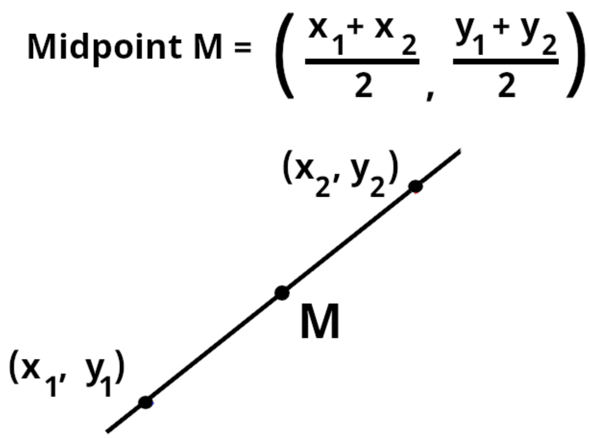

Figure 11.4: The Midpoint Formula

The midpoint between two points is the point in the middle of the line segment between the two points. This point will have coordinates $(X, Y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Find the Midpoint between (1,3) and (4, 7).

$M = (X, Y) = (\frac{1+4}{2}, \frac{3+7}{2})$

$M = (X, Y) = (\frac{5}{2}, \frac{10}{2})$

$M = (X, Y) = (\frac{5}{2}, 5)$

HELPFUL HINT: Students new to this material can confuse the subtraction in the distance formula and the addition in the midpoint formula. Understanding that distance is a difference (in position) and that a midpoint is an average helps keep them distinct.

11.3 The Equation of a Line in the Plane

FACT: The solution to the equation ax + by = c consists of ordered pairs (x, y) the form a line in the xy plane. We often specify the solution by graphing the line.

FACT: In the xy plane, two points determine a line.

We only require two points in the solution to be able to draw the line. Usually this can be done by finding the x and y intercepts of the line. The intercepts refer to where a line crosses the x-axis or the y-axis.

The x-intercept occurs at the point where the value of y is zero. At this point the line intersects the x-axis. The y-intercept occurs at the point where the value of x is zero. Here the line crosses the y-axis.

By setting each variable to zero we can find the x and y intercepts. Drawing the line through the intercepts completes the graph.

EXAMPLE: Graph the solution to the equation 2x + 5y = 20

Many students find it useful to construct a small table. Set x = 0 and solve for y. We get y = 4. Then set y = 0 and solve for x. We get x = 10. This produces the following table.

x	y
0	4
10	0

The table tells us that (0,4) and (10,0) are on the line. We connect the intercepts.

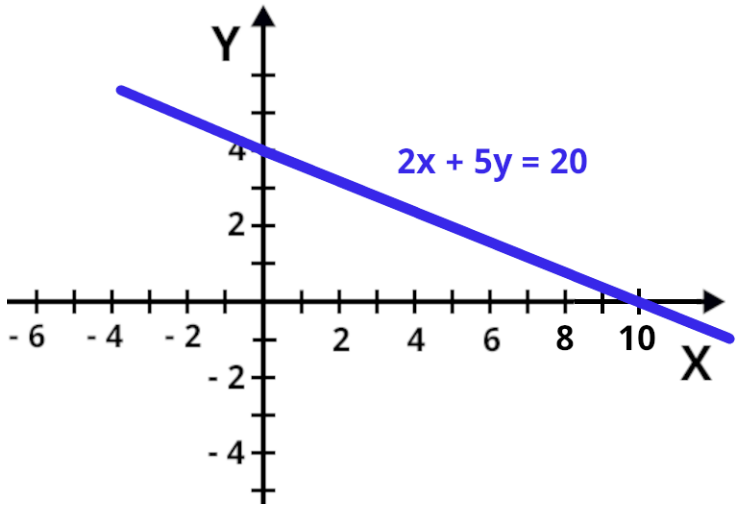

Figure 11.5: Graph of 2x + 5y = 20

Graph the solution to the equation x − 3y = 9

x	y
0	- 3
9	0

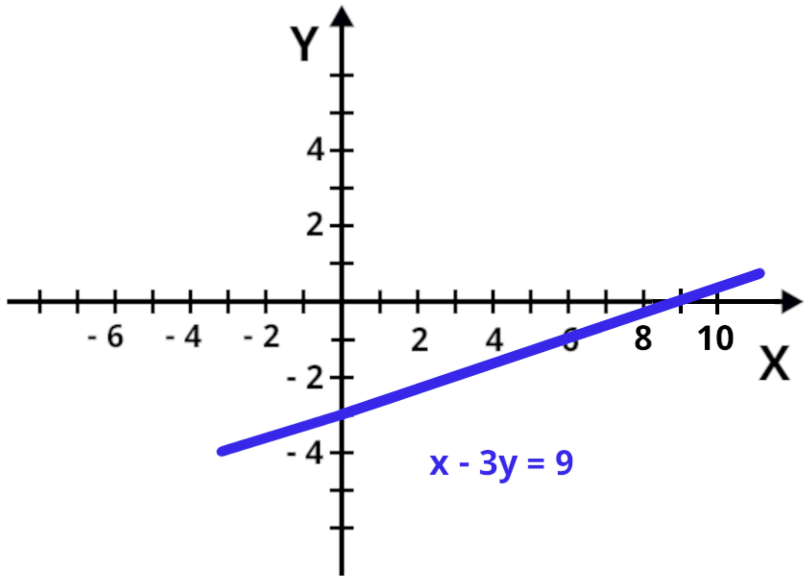

Figure 11.6: Graph of x - 3y = 9

Graph the solution to the equation – 4x + 6y = 24

x	y
0	4
- 6	0

Graph the solution to the equation 4x + 5y = 0

x	y
0	0
0	0

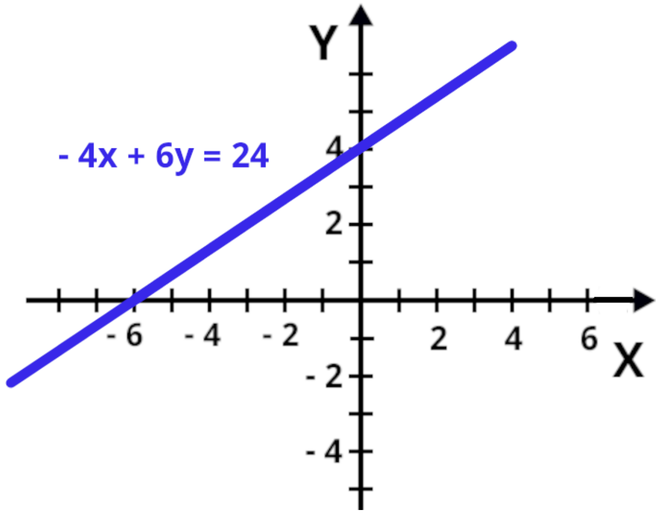

Figure 11.7: Graph of - 4x +6y = 24

Uh oh. Now what? The line passes through the origin (0, 0). The two intercepts are the same point. We require another point. In this case we let either variable equal a value other than zero. With practice one gains a sense of the values that provide integer (not requiring a fraction) coordinates, but letting either variable equal one also provides the required second point. Let x = 1, and get $y = -\frac{4}{5}$. After gaining familiarity with these equations, one would see that setting x = -5 would yield y = 4, giving us a point that is easier to graph.

x	y
0	0
0	0
1	- $\frac{4}{5}$
-5	4

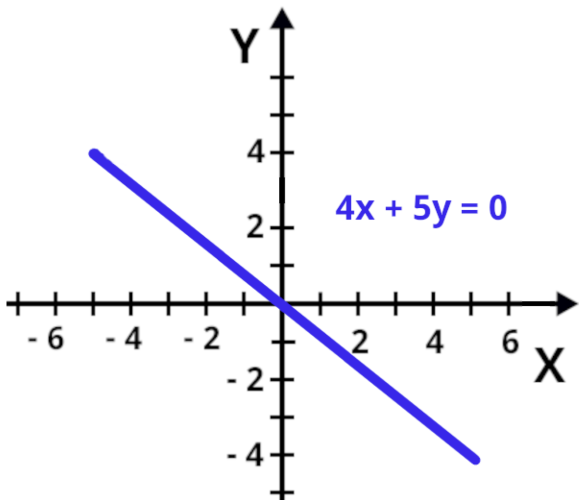

Figure 11.8: Graph of 4x + 5y = 0