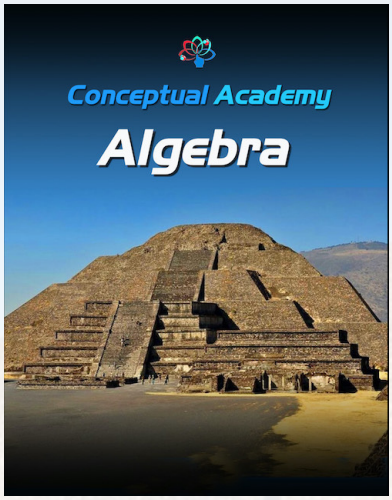
Conceptual Math
Algebra I

Chapter 12: Lines in the Plane



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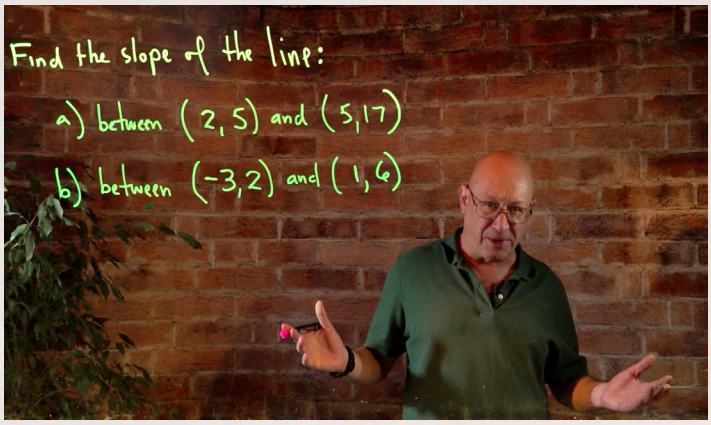

Chapter 12

Lines in the Plane

12.1 Slope and the Slope Intercept Equation

The slope of a line refers to how "steep" it climbs or falls (the change in its y-coordinate) when the x-coordinate is moved to the left or right. More precisely, the slope of a line is equal to the amount that the y coordinate increases (decreases) when the x value increases by 1. If the line is increasing, the slope is positive, and if it is decreasing the slope is negative.

Elementary algebra courses often introduce slope in terms of "rise" (the change in the y coordinate by moving up or down)and "run" (the change in the x coordinate moving left or right.)

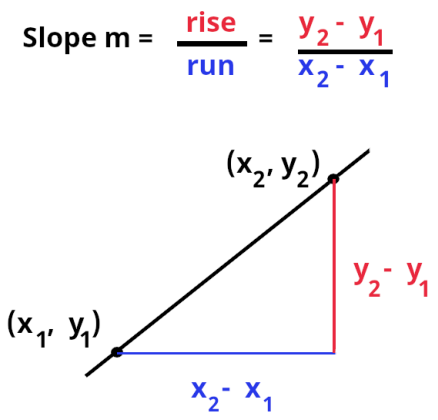

Figure 12.1: The Slope of a Line

Find the slope of the line between (2,5) and (5, 17).

We have $m = \dfrac{17-5}{5-2} = \dfrac{12}{3} = 4.$

Find the slope of the line between (– 3 , 2) and (1, 6).

We have $m = \dfrac{6-2}{1-(-3)} = \dfrac{4}{4} = 1.$

The Slope / Intercept Equation

An equation of the form ax + by = c can be put into the form y = mx + b by solving it for y. When expressed in the form y = mx + b, we know that the slope of the line is m and the y intercept is b.

Returning to $ax + by = c$

Let's solve for y: $by = -ax + c$

So that $y = \left(-\frac{a}{b} \right) x + \frac{c}{a}$

If we let $m = -\frac{a}{b}$ and b $= \frac{c}{a}$, we get what is known as the slope intercept equation:

$y = mx + b$

When written in this form, we know that the slope of the equation is m and the y-intercept of the equation is b.

NOTE: The b in the slope intercept equation is NOT the same number as the b when the equation is written as ax + by = c.

Find the slope and y-intercept of the equation 2x + 5y = 15.

Solve for y to put the equation in slope / intercept form:

$5y = -2x + 15$

$y = -\frac{2}{5}x + 3$

The slope is – 2/5 and the y-intercept is 3.

BEST PRACTICE: When solving for y always put the term with x in front of the constant term so that it easily reads as y = mx + b. Don't put the constant term first.

12.2 The Point Slope Equation

When a line passes through the point (x_1, y_1) with slope m, the equation of the line is:

$y - y_1 = m\ (x - x_1)$

Find the equation of the line through the point (2, 5) with slope 7.

We have y – 5 = 7 (x – 2)

Or y – 5 = 7x – 14

Giving y = 7x – 9

Find the equation of the line through (3, – 2) with slope – 4.

We have y – (– 2) = (– 4) (x – 3)

OR y + 2 = – 4x + 12

Giving y = – 4x + 10.

What is the equation of a line that passes through two given points? We know that the point/slope equation can give us the equation of aline through a point with a given slope. When given two points, we have a point (actually two of them) but not a slope.

The two points provide us a slope, so this becomes a two step process: 1) use the two points to find the slope, and 2) use the point slope equation with either of the two points (take your pick) to find the equation.

Find the equation of the line that passes through (1, – 4) and (3, 2). Specify the slope and the y-intercept.

The slope will be m = $\frac{2-(-4)}{3-1} = \frac{6}{2} = 3$

Now that we have a slope can use either point. Let's pick (3, 2).

From $y - y_1 = m\ (x - x_1)$

We get y – 2 = 3 (x – 3)

y – 2 = 3x – 9

y = 3x - 7

We have the equation of the line in slope intercept form, which tells us that the line has slope 3 and the y-intercept (0, - 7).