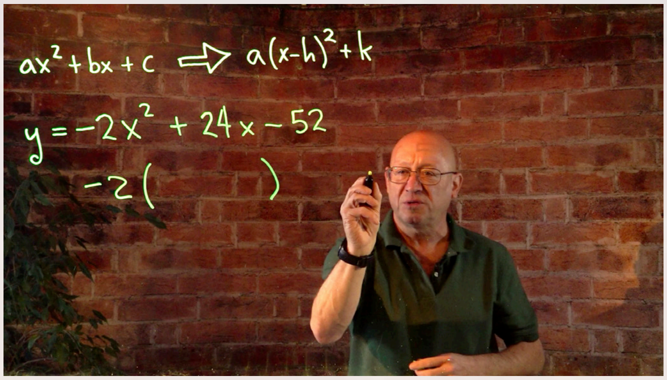

Chapter 15

The Quadratic Function

15.1 The Quadratic Function

When f(x) is a second order polynomial we call it a quadratic function.

A quadratic function has the form f(x) $= ax^2 + bx + c$ where a, b, and c are real numbers. The graph of a quadratic function always takes the form of a parabola. If the coefficient of x^2 is positive, as x increases the parabola will decrease in value until it reaches a minimum and then starts to increase. If the coefficient of x^2 is negative, as x increases the parabola will increase in value until it reaches a maximum and then starts to decrease.

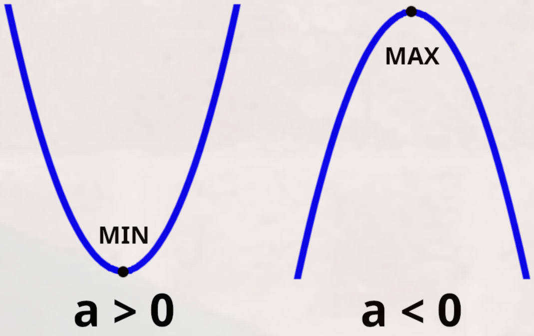

Figure 15.1: Parabola - The Graph of a Quadratic Function

15.2 Vertex Form and Graphing the Parabola

Any quadratic of the form y = $ax^2 + bx + c$ can be converted into the form:

. y = $a(x–h)^2 + k$

This is called the Vertex formula for a quadratic equation. The vertex form is useful because it identifies the vertex of the quadratic's parabola and the shape that it takes.

This quadratic has vertex (h, k).

The coefficient in front of the perfect square tells us the shape of the parabola. It will open upward from the vertex if a is positive. It will open downward if a is negative. It's "steepness" will be the same as the quadratic ax^2.

Any quadratic expressed in this form has a vertex at the point (h, k), and from the value of a we can quickly see how quickly the parabola opens upward or downward from that vertex.

When given a quadratic in the form ax2 + bx + c, it is always possible to convert the expression into the vertex form of the quadratic. The process that does this is called Completing the Square.

NOTE: The vertex form of a quadratic contains the perfect square of (x – h)^2.

The quadratic expressed as $ax^2 + bx + c$, does not.

KEY CONCEPT: We are going to change the FORM of the expression without changing what it is mathematically saying at all. The two expressions will be equal, but the way one is expressed allows us to see information that the other does not.

Consider the quadratic $x^2 + 6x + 5$

We want to express this in way that has a perfect square of (x + ?) 2 .

What would we add to the x in (x +) 2 that would give us x2 + 6x as the first two terms?

NOTE: You will have to be comfortable with the perfect squares of

$(x+1)^2 = x^2 + 2x + 1$ $(x–1)^2 = x^2–2x + 1$

$(x+2)^2 = x^2 + 4x + 4$ $(x–2)^2 = x^2–4x + 4$

$(x+3)^2 = x^2 + 6x + 9$ $(x–3)^2 = x^2–6x + 9$

$(x+4)^2 = x^2 + 8x + 16$ $(x–4)^2 = x^2–8x + 16$

$(x+5)^2 = x^2 + 10x + 25$ $(x–5)^2 = x^2–10x + 25$

$(x+6)^2 = x^2 + 12x + 36$ $(x–6)^2 = x^2–12x + 36$

To become comfortable with completing the square.

Let's complete the square for the quadratic $x^2–6x + 10$

$(x^2 \; - \; 6x \qquad) \qquad + 10$

$(x^2 - 6\text{x} + 9\ ) \quad - 9 \quad + 10$

$(x - 3)^2 \;\; + 1$

This tells us the the vertex is (3, 1) and that it opens upward with the shape of x^2.

We can now graph it.

Let's complete the square for the quadratic x^2 + 12x + 33

$(x^2 \; + \; 12x \qquad) \qquad + \quad 33$

$(x^2 \; + \; 12x \;\; + \; 36) \;\; - \;\; 36 \quad + \quad 33$

$(x + 6)^2 - 3$

This tells us the vertex is $(-6, -3)$ and that it opens upward with the shape of x^2.