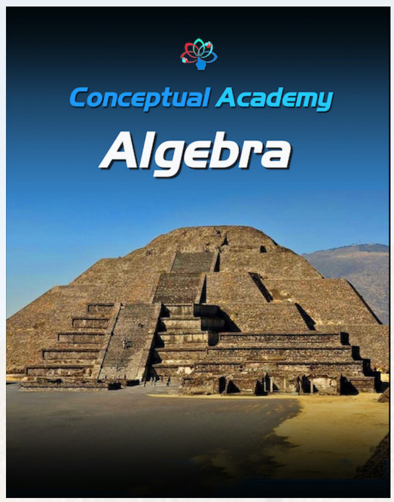
Conceptual Math
Algebra I

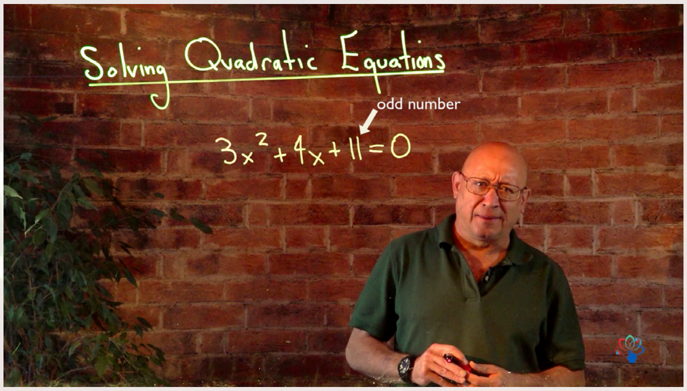
Chapter 16: Quadratic Equations



Matt Foraker, Ph.D.
Western Kentucky University
Bowling Green, KY

Conceptual Academy

All inquiries
Support@ConceptualAcademy.com



Chapter 16

Quadratic Equations

A quadratic equation has the form $ax^2 + bx + c = 0$.

We may encounter a quadratic equation that is not set to zero, such as $3x^2 = 11x + 4$. When this occurs, one sets it equal to zero by adding or subtracting until the right hand side of the equation is zero. In this case we would get $3x^2 - 11x + 4 = 0$.

16.1 Factoring and Completing the Square

If we can factor the quadratic, we can use the principle of zero products to solve the equation.

The principle of Zero Products:

If AB = 0, then either A = 0 or B = 0 or both A and B are zero. In the context of quadratics:

If QUADRATIC = (Factor A)(FACTOR B) THEN:

FACTOR A = 0 will be a solution.
FACTOR B = 0 will be a solution.

Conisder our example above: $3x^2 - 11x + 4 = 0$

This can be factored to (3x + 1)(x - 4). Set each of these to zero.

3x + 1 = 0. Solve for x and obtain x = $-\frac{1}{3}$.
x - 4 = 0. Solve for x and obtain x = 4.

The solution to the equation is x = 4 and $-\frac{1}{3}$.

Solve $x^2–5x + 6 = 0$.

We can factor this to (x – 3) (x – 2) = 0.

So we get x = 3 and x = 2.

Solve: $x^2 = 10 - 3x$.

Set equal to zero: $x^2 + 3x–10 = 0$.

Factor: (x + 5) (x – 2) = 0.

So we have x = – 5 and 2.

Many quadratics will not factor. They can still be solved with other methods.

FACT: The quadratic equation $ax^2 + bx + c = 0$, can be solved for x in terms of a, b, and c. This produces one of the most important results in college algebra, the quadratic formula.

16.2 The Quadratic Formula

The equation $ax^2 + bx + c = 0$ has the following solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There is no quadratic equation that cannot be solved with the quadratic formula.

Example: Solve $x^2 – 7x + 10 = 0$.

We have x $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Or x $= \dfrac{7 \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$

x $= \dfrac{(7 \pm \sqrt{9})}{2}$

x = 5, 2

Example: Solve $2x^2 + 4x = 15$

Set equal to zero. $2x^2 + 4x – 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 – 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 – 4(1)(-15)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{136}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{34}}{4}$$

$$x = \frac{-2 \pm \sqrt{34}}{2}$$

Solve $3x^2 - 12 = 16x$

Set equal to zero: $3x^2 – 16x - 12 = 0$

$$x = \frac{16 \pm \sqrt{16^2 – 4(3)(-12)}}{2(3)}$$

$$x = \frac{16 \pm \sqrt{256 + 144}}{6}$$

$$x = \frac{16 \pm \sqrt{400}}{6}$$

$$x = \frac{16 \pm 20}{6}$$

$$x = \frac{36}{6}\ ,\ \frac{-4}{6}$$

$$\text{x} = 6,\ -\frac{2}{3}$$

Solve $3x^2 + 4x + 11 = 0$

$$x = \frac{-4 \pm \sqrt{(-4)^2 – 4(3)(11)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{16 – 132}}{6}$$

$$x = \frac{-4 \pm \sqrt{-16}}{6}$$

$$x = \frac{-4 \pm 4i}{6}$$

$x = \dfrac{-2 \pm 2i}{3}$

16.3 The Discriminant

The expression $b^2 - 4ac$ in the quadratic formula is known as the discriminant. Note that the discriminant refers to what is under the radical sign. It does not include the radical.

The sign of the discriminant provides valuable information regarding the nature of the quadratic function and the solution when it is set to zero.

Case 1: The discriminant $b^2 - 4ac > 0$

When $b^2 - 4ac > 0$ we will have two real solutions to the equation. The pqrabola that graphs the quadratic will intersect the x-axis at the x values that solve the equation.

Case 2: The discriminant $b^2 - 4ac = 0$

When $b^2 - 4ac = 0$ we will have one real solution to the equation. The parabola that graphs the quadratic intersects the a-axis precisely at its vertex.

Case 3: The discriminant $b^2 - 4ac < 0$

When $b^2 - 4ac < 0$ there are no real solutions to the equation. We will have two solutions, but they will be complex. The parabola that graphs the quadratic DOES NOT intersect the x-axis.