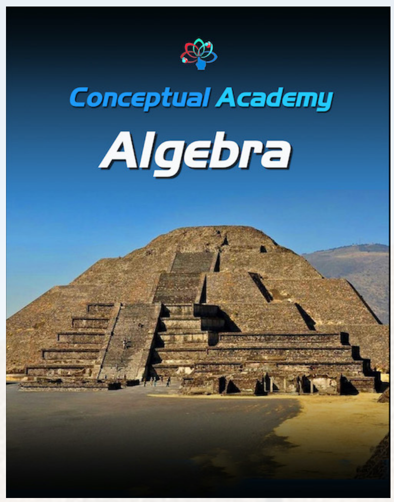
Conceptual Math
Algebra I

Chapter 17: Applications of Quadratic Functions



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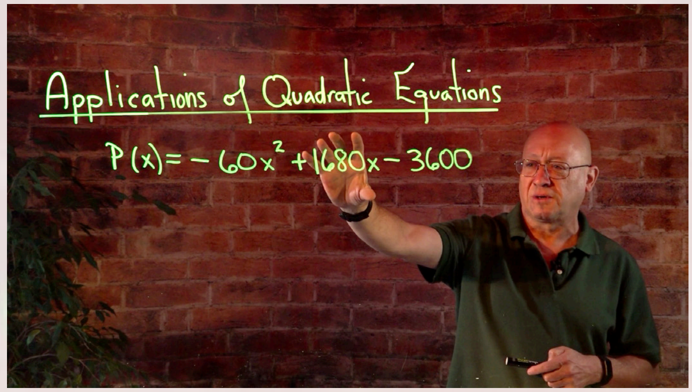

Chapter 17

Quadratic Applications

Situations occur in geometry, physics, finance, and other fields that can be expressed or modeled as quadratic functions. When we have a quadratic function in an application, we are typically asked one or more of the following:

1. At what value of the input to the function does the output equal a particular value? For example, if the height of a projectile is given by h(t) = $-16t^2 + 240t + 1200$ where h(t) is in feet and t is in seconds, at what time(s) is the height equal to 1000 ft?

2. At what value of the input does the value of a function reach its maximum or minimum and what is that maximum or minimum? For example, if the cost per toolkit in dollars is given by c(x) = $0.05x^2 + 40x + 225$, how many toolkits should be purchased for the lowest cost per kit? What is that cost?

17.1 Some Geometry

Geometric calculations can involve quadratic functions.
Example: A rectangular fence is to be constructed against the back of a school building to enclose a playground. The back of the school is a straight wall, and they have 500 feet of fence. What is the maximum area possible for the playground? What should its dimensions be?
Solution: The area of a rectangle is length times width. If we let x = the

length of the fence coming out from the school wall to the outer end of the playground, we know that the length of the outer eddge will be 500 - 2x. The area of the playground = x (500 - 2x).

Area = $500x - 2x^2$

If we set this to zero, we get x = 0 and x = 250. The value of zero is placing all 500 feet of the fence directly against the school wall. The value of 250 is putting two pieces of 250 against each other straight from the school. Both provide a playground with zero area.

This is true, but it has no value and doesn't answer the question asked. What dimensions provide the MAXIMUM area.

To answer questions of maximum or minimum, we put the quadratic into vertex form.

A(x) = $-2x^2 + 500x$

A(x) = $-2(x^2 - 250x\ )$

A(x) = $-2(x^2 - 250x \ \ + \ 15625) \ + \ \ 31,250$

A(x) = $-2(x - 125)^2 \ + 31,625$

Now that we have the function in vertex form, we can see that the maximum area = 31,625 square feet when the walls from the school are 125 feet in length.

17.2 Cost and Profit Functions

COST FUNCTION

Sally runs a shoe store and buys bulk shoes at Acme Shoe Warehouse. If her cost in dollars per pair is given by c(x) = $0.1x^2 – 8x + 230$ when she buys x

pairs of shoes, how many pair should she buy to minimize her cost per pair? What is that minimum cost?
The cost function is given by $c(x) = 0.1x^2 – 8x + 230$
To find the minimum cost, we require the vertex of the graph of the quadratic. We do this by completing the square and putting the quadratic into the vertex form of $a(x–h)^2 + k$.

Complete the square.

We have $0.1(x^2–80x \quad\quad) \quad\quad + 230$

Or $0.1(x–40)^2 + 230$

Or $0.1(x^2–80x + 1600)–160 + 230$

Or $0.1(x–40)^2–160 + 230$

Or $0.1(x–40)^2 + 70$

Were we to graph this, we would have vertex at (40, 70) and shape of $0.1x^2$.

We have h = 40 and k = 70.

She should buy 40 pairs of shoes and the cost per pair is \$ 70.

PROFIT FUNCTION

A computer manufacturing company produces and sells laptops. Their profit in in thousands of dollars is given by:

P(x) = $-60x^2 + 1680x–3600$ where x is the number of laptops sold (in thousands).

How many laptops should they sell to maximize their profit and what would the maximum profit be?

We require the vertex form of the function. Complete the square.

P(x) = –60 (x^2 –$28x$) – 3600

P(x) = –60 (x^2 – $28x$ + 196) + 11760 – 3600

P(x) = –60 $(x - 14)^2$ + 8160

The vertex is (14, 8160)

They should sell 14,000 laptops to earn a maximum profit of $ 8,160,000.

17.3 Projectiles

An object launched or dropped into the air has an altitude given by the equation:

$h(t) = –16t^2 + v_0t + h_0$

where t is in seconds, v_0 is the initial vertical velocity and h_0 is the original height.

Example: Bob shoots a tennis ball into the air with his homemade tennis ball cannon. The altitude of the ball in feet at time t is given by $h(t) = –16t^2 + 640t$. What maximum height did the ball reach and how long did it take before the ball returns to the ground?

What is its maximum height?

We have $h(t) = –16t2 + 640t$

Must put into vertex form to find maximum value.

Complete the square $–16t^2 + 640t$

$–16(t^2–40t \quad)$

$–16(t–20)^2$

$–16(t^2–40t + 400) + 6400$

$–16(t–20)^2 + 6400$

The vertex is (20, 6400)

The ball reaches a maximum height of 6400 feet in 20 seconds.

To find when it hits the ground, recognize that this occurs when h(t) = 0.

OR $–16t^2 + 640t = 0$

Note that we can simplify the calculations by dividing by negative 16.

$t^2–40t = 0$.

Or t (t – 40) = 0.

We get the solutions t = 0 and t = 40.

The ball hits the ground after 40 seconds.

The equation is telling us that the ball is on the ground at t = 0 and at t = 40.

Does that make sense?

Draw the parabola.

Example: Bob shoots a tennis ball over a 1200-foot cliff overlooking the ocean below. If the height of the tennis ball over the ocean is given by h(t) = $–16t^2+960t+1200$, where h(t) is in feet and t is in seconds, find the maximum height reached by the ball. How long does it take to fall into the ocean below?

This is quadratic whose maximium / minimum is given by the (h,k) when the function is expressed in vertex form. Complete the square for vertex form.

h(t) = − 16 $(t^2 - 60t) + 1200$

h(t) = − 16 $(t^2 - 60t + 900) + 14,400 + 1200$

h(t) = − 16 $(t - 30)^2$ + $15,600$

(h, k) = (30, 15,600)

At t = 30 seconds, the ball reaches maximum height of 15,600 feet.

To determine when the ball hits the ocean, recognize that the ball has a height of 0 feet when this occurs. Set h(t) to zero.

$h(t) = -16t^2 + 960t + 1200 = 0.$

This is just a quadratic equation that is easily solved using the quadratic formula.

Since this is an equation, we can divide both sides by any number, so if there is a common factor in the terms, we can divide by this to get smaller numbers to put into the formula.

We can divide the whole equation by 16.

$h(t) = -t^2 + 60t + 75 = 0.$

By the quadratic formula,

$$t = \frac{-60 \pm \sqrt{(60)^2 - 4(-1)(75)}}{2(-1)}$$

$$t = \frac{-60 \pm \sqrt{3600 + 300}}{-2}$$

$$t = \frac{-60 \pm \sqrt{3900}}{-2}$$

$$t = \frac{-60 \pm 62.45}{-2}$$

t = – 1.225 seconds and 61.225 seconds.

What is happening? Negative time? The positive answer works. The ball will hit the ground in 61.2 seconds, but what is that negative result doing there?

THE SHORT ANSWER – While Physics knows Algebra, Algebra does not know Physics.

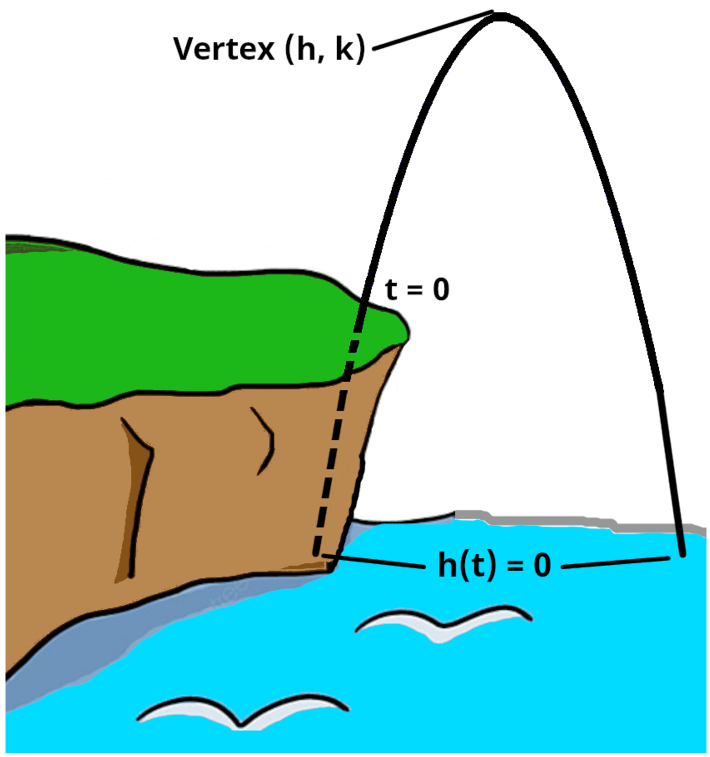

Figure 17.1: Shooting a Projectile over a Cliff

THE LONGER ANSWER – The formula for the height of the ball is correct, but it doesn't have a problem with negative time. Time is just another number. If we graph h(t), we get a parabola with a maximum at (30, 15600). At t=0, the ball is at 1200 feet over the ocean with Bob. If we were to go back in time, the graph hits the x-axis at t = minus 1.225 seconds.