Chapter 1

About Science

THE MAIN IDEA

Science is the study of nature's rules

1.7 Scientific Notation

In science, we often encounter very large and very small numbers. Written in standard decimal notation, these numbers can be cumbersome. There are, for example, about 33,460,000,000,000,000,000,000 water molecules in a thimbleful of water, each having a mass of about 0.00000000000000000000002991 gram. To represent such numbers, scientists use a mathematical shorthand called **scientific notation**. Written in this notation, the number of molecules in a thimbleful of water is 3.346×10^{22} and the mass of a single molecule is 2.991×10^{-23} gram.

To understand how this shorthand notation works, consider the large number 50,000,000. Mathematically, this number is equal to 5 multiplied by $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. (Check this out on your calculator.) We can abbreviate this chain of numbers by writing all the 10s in an exponential form, which gives us the scientific notation 5×10^7. (Note that 10^7 is the same as $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$.) **Table 1.4** shows the exponential form of some other large and small numbers.

TABLE 1.4 Large and small numbers. Note: All of these numbers are greater than zero.

1,000,000	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$
100,000	$= 10 \times 10 \times 10 \times 10 \times 10 = 10^5$
10,000	$= 10 \times 10 \times 10 \times 10 = 10^4$
1000	$= 10 \times 10 \times 10 = 10^3$
100	$= 10 \times 10 = 10^2$
10	$= 10 = 10^1$
1	$= 1 = 10^0$
0.1	$= 1/10 = 10^{-1}$
0.01	$= 1/(10 \times 10) = 10^{-2}$
0.001	$= 1/(10 \times 10 \times 10) = 10^{-3}$
0.0001	$= 1/(10 \times 10 \times 10 \times 10) = 10^{-4}$
0.00001	$= 1/(10 \times 10 \times 10 \times 10 \times 10) = 10^{-5}$
0.000001	$= 1/(10 \times 10 \times 10 \times 10 \times 10 \times 10) = 10^{-6}$

Likewise, the small number 0.0005 is mathematically equal to 5 divided by 10 × 10 × 10 × 10, which is $5 \div 10^4$. Because dividing by a number is equivalent to multiplying by the reciprocal of that number, $5 \div 10^4$ can be written in the form 5×10^{-4}. So, in scientific notation 0.0005 becomes 5×10^{-4}. (Note the negative exponent.)

All scientific notation is written in the general form:

$$C \times 10^n$$

where C, called the coefficient, is a number less than 10 but equal to or greater than 1 (in other words, between 1 and 9.999 …) and n is the exponent. A positive exponent indicates a number greater than 1, and a negative exponent indicates a number between 0 and 1 (not a number less than 0). Numbers less than 0 are indicated by putting a negative sign before the coefficient (not in the exponent). Some common examples are shown in **Table 1.5**.

To change a decimal number that is greater than +1 or less than -1 to scientific notation, you shift the decimal point to the left until you arrive just before the first digit. For example, to convert the decimal number 45,000 to scientific notation, move the decimal point four places to the left:

$$45{,}000 = 4.5 \times 10^4$$
$$4321$$

For decimal numbers between +1 and -1, you move the decimal point to the right until you arrive just before the last digit. This number is the coefficient part of the notation. The exponent part is simply the number of places you moved the decimal point. The number 0.00045, for example, is converted to scientific notation by moving the decimal point four places to the right:

$$0.00045 = 4.5 \times 10^{-4}$$
$$1234$$

Note that because you moved the decimal point to the right in this case, you must put a minus sign on the exponent.

TABLE 1.5 Numbers in both decimal and scientific notation

	DECIMAL NOTATION	SCIENTIFIC NOTATION
Large positive number (greater than 1)	6,000,000,000	6×10^9
Small positive number (between 0 and 1)	0.0006	6×10^{-4}
Large negative number (less than −1)	−6,000,000,000	-6×10^9
Small negative number (between −1 and 0)	−0.0006	-6×10^{-4}

TABLE 1.6 Some examples of physical data using scientific notation

Number of molecules in thimbleful of water	3.346×10^{22}
Mass of water molecule	2.991×10^{-23} *gram*
Average radius of hydrogen atom	5×10^{-11} *meter*
Proton mass	1.6726×10^{-27} *kilogram*
Neutron mass	1.6749×10^{-27} *kilogram*
Electron mass	9.1094×10^{-31} *kilogram*
Electron charge	1.602×10^{-19} *coulomb*
Avogadro's number	6.022×10^{23} *particles*
Atomic mass unit	1.661×10^{-24} *gram*

CONCEPT **CHECK**

Express the following exponentials as decimal numbers:

a. 1×10^{-7}

b. 1×10^{8}

c. 8.8×10^{5}

Express the following decimal numbers in scientific notation:

d. 740,000

e. -0.00354

f. 15

CHECK YOUR ANSWER

a. 0.0000001 b. 100,000,000 c. 880,000 d. 7.4×10^{5} e. -3.54×10^{-3} f. 1.5×10^{1}